

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics – I

Subject Code : 4TE01EMT1

Branch: B.Tech (All)

Semester : 1

Date : 21/03/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions:

(14)

- a) The points representing the complex number z for which $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$ lie on
(A) a circle (B) a straight line (C) an ellipse (D) a parabola
- b) If $\left(\frac{3}{2} + \frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}(x-iy)$ where x, y are real, then the ordered pair (x, y) is given by
(A) $(0, 3)$ (B) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ (C) $(-3, 0)$ (D) $(0, -3)$
- c) If $f(x) = \frac{e^x - e^{-x}}{2}$ is continuous at $x=0$, then the value of $f(0)$ must be
(A) 0 (B) 1 (C) 2 (D) 3
- d) $\lim_{x \rightarrow \infty} \frac{x^2}{e^x} = \underline{\hspace{2cm}}$
(A) -1 (B) 0 (C) 1 (D) none of these
- e) The infinite series $1+r+r^2+\dots+r^{n-1}$ is divergent if
(A) $|r| < 1$ (B) $|r| > 1$ (C) $r \geq 1$ (D) $r = -1$
- f) The series $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots \infty$ is
(A) convergent (B) divergent (C) absolutely convergent (D) none of these
- g) If the two tangents at the point are imaginary, the double point is called
(A) a node (B) a cusp (C) a conjugate point (D) none of these
- h) The curve $x^3 + y^3 = 3axy$ represent
(A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid
(D) Folium of Descartes



- i) The series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$ represent expansion of
 (A) $\sin x$ (B) $\cos x$ (C) $\sinh x$ (D) $\cosh x$
- j) If $y = \cos^{-1} x$, then x equal to
 (A) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ (B) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$ (C) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!}$
 (D) none of these
- k) Conditions for $f(x, y)$ to be maximum are
 (A) $f_x = 0 = f_y, rt < s^2, r < 0$ (B) $f_x = 0 = f_y, rt > s^2, r < 0$
 (C) $f_x = 0 = f_y, rt > s^2, r > 0$ (D) $f_x = 0 = f_y, rt = s^2, r > 0$
- l) If $u = y^x$, then $\frac{\partial u}{\partial x}$ is
 (A) xy^{x-1} (B) 0 (C) $y^x \log x$ (D) none of these
- m) $\frac{\partial(u, v, w)}{\partial(x, y, z)} \cdot \frac{\partial(x, y, z)}{\partial(u, v, w)} = \text{_____}$
 (A) 2 (B) 1 (C) 0 (D) none of these
- n) If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to
 (A) $\frac{\partial f / \partial x}{\partial f / \partial y}$ (B) $\frac{\partial f / \partial y}{\partial f / \partial x}$ (C) $-\frac{\partial f / \partial y}{\partial f / \partial x}$ (D) $-\frac{\partial f / \partial x}{\partial f / \partial y}$

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) Prove that $(a + ib)^{\frac{m}{n}} + (a - ib)^{\frac{m}{n}} = 2(a^2 + b^2)^{\frac{m}{2n}} \cos\left(\frac{m}{n} \tan^{-1} \frac{b}{a}\right)$. (5)
- b) Using Sandwich theorem show that (5)
 (i) $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ (ii) $\lim_{x \rightarrow 0} x^2 \cos 20\pi x = 0$
- c) Evaluate: $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ (4)

Q-3 Attempt all questions (14)

- a) Using De Moivre's theorem prove that $\sin 5\theta = 5\sin \theta - 20\sin^3 \theta + 16\sin^5 \theta$. (5)
- b) Prove that $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right) = \frac{1}{2}$. (5)
- c) Find the roots of the equation $x^6 - i = 0$. (4)

Q-4 Attempt all questions (14)

- a) Expand $f(x) = \frac{e^x}{e^x + 1}$ in powers of x up to x^3 by Maclaurin's series. (5)
- b) Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$ (5)



c) Test for convergence the series $\sum \sin\left(\frac{1}{n}\right)$. (4)

Q-5 Attempt all questions (14)

a) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is (i) convergent if $p > 1$ and (ii) divergent if $p \leq 1$. (5)

b) Test for convergence the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{x}{4 \cdot 5 \cdot 6} + \frac{x^2}{7 \cdot 8 \cdot 9} + \dots$ (5)

c) Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem. (4)

Q-6 Attempt all questions (14)

a) If $u = \sec^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$ then prove that (5)

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\cot u (\cot^2 u + 2).$$

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then verify $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}$. (5)

c) Find the asymptotes of the curve $x^2 y^2 - x^2 y - 5xy^2 + x + y + 5 = 0$. (4)

Q-7 Attempt all questions (14)

a) Trace the curve $r^2 = a^2 \cos 2\theta$. (5)

b) Discuss the maxima and minima of $xy + 27\left(\frac{1}{x} + \frac{1}{y}\right)$. (5)

c) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ then verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. (4)

Q-8 Attempt all questions (14)

a) Trace the curve $y^2(2a - x) = x^3$. (5)

b) The power consumed in an electric resistor is given by $P = \frac{E^2}{R}$ (in watts). If $E = 200$ volts and $R = 8$ ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0.20 ohms? (5)

c) If $u = 2xy$, $v = x^2 - y^2$ and $x = r \cos \theta$, $y = r \sin \theta$ then evaluate $\frac{\partial(u, v)}{\partial(r, \theta)}$. (4)

